MCQ of Chapter 4 (Sequence)

1. If $x_n = \sin\frac{n\pi}{2}$, $n \in \mathbb{N}$, then
   a) $\{x_n\}_n$ is a bounded sequence
   b) $\{x_n\}_n$ is an unbounded sequence
   c) $\{x_n\}_n$ is a convergent sequence
   d) $\{x_n\}_n$ is not a sequence

2. The sequence $\left\{\frac{7n}{n+3\sqrt{n}}\right\}$
   a) converges to $\frac{7}{4}$ b) converges to 7 c) converges to $\frac{7}{3}$ d) is not convergent

3. The sequence $\{x_n\}_n$, defined by $x_1 = 1$, $x_{n+1} = \frac{4+3x_n}{3+2x_n}$, $n \in \mathbb{N}$
   a) converges to $\frac{4}{3}$ b) converges to 2 c) converges to 1 d) converges to $\sqrt{2}$

4. $\lim_{n\to\infty} \frac{1}{n} \sum_{1}^{n} \sqrt[n]{n}$
   a) equals to 0 b) does not exist c) equals to 1 d) equals to e

5. The sequence $\{x_n\}$ defined by $x_{n+1} = \sqrt{3x_n}$, $n > 1$ and $x_1 = \sqrt{3}$ converges to
   a) 3 b) $\sqrt{3}$ c) $3\sqrt{3}$, d) $\sqrt{3\sqrt{3}}$

6. $\lim_{n\to\infty} \frac{\sin\left(n\frac{\pi}{3}\right)}{\frac{n}{3}}$
   a) equals to π b) equals to 0 c) equals to 1 d) does not exist

7. If $x_n = \frac{n!}{n^n}$, then $\lim_{n\to\infty} x_n$ equals to
   a) e b) $\frac{1}{e}$ c) 1 d) 0

8. If $x_n = \frac{\sqrt[n]{n!}}{n}$, then $\lim_{n\to\infty} x_n$
a) equals to e b) does not exist c) equals to $\frac{1}{e}$ d) equals to 0

9. The sequence $\{x_n\}$ is defined by $x_{n+1} = x_n(2 - x_n)$, $x_n > 0 \ \forall n \in \mathbb{N}$. If $0 < x_1 < 1$, then $\{x_n\}$ is
a) bounded below only b) monotone decreasing
c) not convergent d) none of these

10. The sequence $\{x_n = n^{(-1)^n}\}_{n\in\mathbb{N}}$
a) is convergent b) has both convergent and divergent subsequences
c) is bounded d) None of these

11. The sequence $\{x_n = \sin\frac{n\pi}{2}\}_{n\in\mathbb{N}}$
a) is bounded b) is convergent
c) has only convergent subsequences converging to 0 and 1
d) None of these

12. lim sup and lim inf of the sequence $\{(-1)^n + \frac{1}{n}\}$ are respectively
a) 1, −1 b) 1, 0 c) $\frac{3}{2}$, 0 d) 0, −1

13. A sequence $\{x_n\}$ whose sup, inf, lim sup and lim inf are all distinct is
a) $(-1)^n$ b) $n(-1)^n$ c) $\frac{(-1)^n}{n}$ d) $x_n = \begin{cases} 2-\frac{1}{n}, & n \text{ is odd} \\ 3+\frac{1}{n^2}, & n \text{ is even} \end{cases}$

14. The sequence $\{x_n\}$, where $x_n = \left(1+\frac{2}{n}\right)^{n+3}$ converges to
a) e b) e+3 c) e^2 d) e^2+3

15. Which of the following statements is true?
a) Convergent sequences are monotonic.
b) Monotonic sequences are convergent.

c) Every sequence has a convergent subsequence

d) Convergent sequences are Cauchy sequence [C.H. 2019]

16. Lim sup of the sequence $\left\{\frac{1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}}{n}\right\}_{n=1}^{\infty}$ is

a) 0 b) 1 c) ∞ d) $\frac{1}{\sqrt{2}}$ [C.H. 2019]

17. If $0<x<2021$, then

$$\lim_{n\to\infty} \frac{x^{n+1}+2021^{n+1}}{x^n+2021^n}$$ is

a) 0 b) x c) 2021 d) 1 [C.H. 2021]

18. If $x_n = \frac{1+\sin^2(n^2+1)\pi + \cos(n^3-5)\pi}{n+1}$, then lim sup x_n is

a) 3 b) 2 c) 1 d) 0 [C.H. 2021]

19. Identify the incorrect statement from the following;

a) Every convergent sequence is bounded

b) Every bounded sequence has at least one subsequential limit

c) Unbounded sequence can not have any subsequential limit

d) Unbounded sequence is never a Cauchy sequence [C.H. 2021]